TRINITY COLLEGE

Semester One Examination, 2023

Question/Answer booklet

MATHEMATICS METHODS UNIT 3

If required by your examination administrator, please place your student identification label in this box

Section One: Calculator-free

WA student number: In figures | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	12	12	100	100	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **35% (50 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

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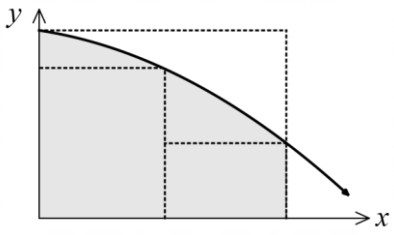
See next page

Question 1 **(6 marks)**

The curve $y = 15 - 2x - x^2$ is shown, with a bounding rectangle and two inscribed rectangles of equal width.



The shaded region is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

(a) Use areas of rectangles to explain why the area of the shaded region must be between 19 and 30 square units. (3 marks)

(b) Determine the area of the shaded region. (3 marks)

Question 2 **(8 marks)**

The probability function for the random variable X is $\mathrm{P}(X = x) = \begin{cases} 10k^2 + kx, & x = 0 \\ 5k^2 - 2kx, & x = 1 \\ 0, & \text{otherwise.} \end{cases}$

(a) Determine the value of the constant k. (4 marks)

(b) Determine the mean and variance of X. (2 marks)

(c) The random variable $Y = 5X - 1$. Determine the mean and variance of Y. (2 marks)

See next page

Question 3 **(9 marks)**

The function $f(x)$ is defined for $x > 1.5$, has derivative $f'(x) = \frac{8}{(2x-3)^2}$, and passes through the point $(4, 1)$.

(a) Determine the rate of change of $f'(x)$ when $x = 3$. (3 marks)

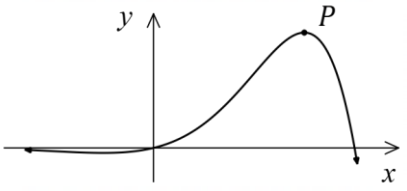
(b) Determine $f(x)$. (4 marks)

(c) Determine $\frac{d}{dt}\int_t^3 (f'(x) - 4x)\,dx$. (2 marks)

See next page

Question 4 **(8 marks)**

The graph of $y = e^{5x}\sin(5x)$ is shown.



(a) Determine the x-coordinate of point P, the first local maximum of the curve as x increases from 0. (4 marks)

(b) Determine the value of $\frac{d^2y}{dx^2}$ when $x = \frac{4\pi}{5}$ and hence describe the concavity of the curve at this point. (4 marks)

See next page

Question 5 **(6 marks)**

(a) Determine $f'(x)$ when $f(x) = \dfrac{4 + \cos(x)}{4 + \sin(3x)}$. There is no need to simplify the derivative.

(2 marks)

(b) Let $y = \cos(x)$, so that when $x = 150°,\ y \approx -0.8660$. Given that $1° \approx 0.017$ radians, use the increments formula to determine an approximate value for $\cos(151°)$. (4 marks)

See next page

Question 6 **(7 marks)**

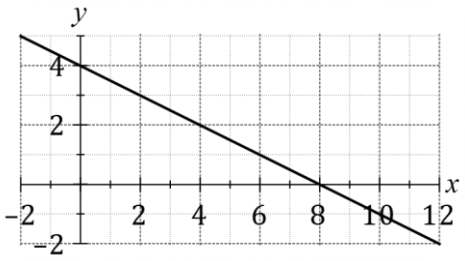
An 8 cm length of thin straight wire is bent once and laid on a level surface to form side BC and diagonal CE of rectangle $BCDE$. Let the length of $BC = x$.

(a) Show that the area of the rectangle is $x\sqrt{64 - 16x}$ cm^2. (3 marks)

(b) Determine the maximum possible area of the rectangle. (4 marks)

Question 7 **(6 marks)**

The graph of the linear function $y = f(x)$ is shown.

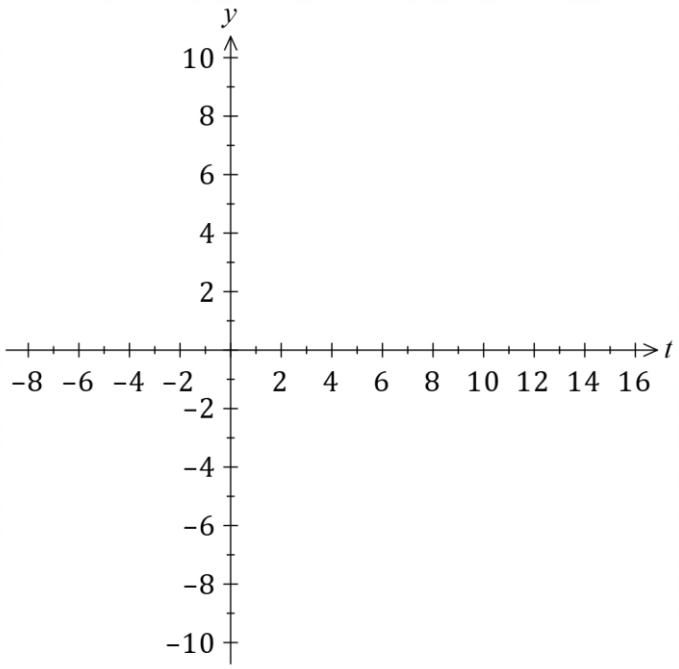


Another function is defined as

$$A(t) = \int_2^t f(x)\,dx$$

(a) Using the graph of $y = f(x)$, or otherwise, evaluate $A(2)$ and $A(8)$. (2 marks)

(b) Hence, or otherwise, sketch the graph of $y = A(t)$ on the axes below. (4 marks)



End of questions

Supplementary page

Question number: _____